Parallel 1

MATHEMATICS: ARE YOU SPEAKING MY LANGUAGE?

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ABSTRACT

What is the role of mathematics language in learning mathematics? Mathematics language is viewed as a medium through which mathematics content is communicated. But mathematics is no one's native language, and so no one thinks or communicates totally in mathematics. Yet, more than any other discipline, mathematics requires careful translation, as much as any foreign language does. If translation breaks down, misconceptions grow and mathematical thinking suffers. Mathematics language is precise and frequently contains high concepts density; a few words and symbols often convey a very complex idea. To most of us, the language of mathematics is like a foreign language, that is a second language with its own grammar, syntax, vocabulary, word order, negation, conventions, and sentence structure. As teacher, educator, or mathematician, we often overlook the difficulties faced by our students in understanding letters and words, in interpreting mathematical sentences and paragraphs or in understanding the process of problem solving. In this presentation, the presenter will share about language concepts of mathematics, difficulties that arise in the translation of mathematics, and the implications for teaching and learning of mathematics.

Introduction

The language used to convey mathematical ideas to students has become a topic of increased concern to mathematics educators in recent years. In the United States this concern has risen in part because of a continuing growth in the number of students in mathematics classes who have a limited proficiency in English. An inadequate grasp of the language of instruction is a major source of underachievement in school.

This paper explores issue for research and instruction arising from the learning of mathematics by students for whom English is a second language. After some important aspects of the process of second language acquisition have been identified, two questions are addressed: To what extent are English and the student's native language critical factors in learning mathematics. And, how is the assessment of mathematics achievement related to second-language skills? The paper concludes with a review of issues in mathematics instruction when English is the students' second language. Mathematic educators working with students whose native language is not English need to be more cognizant of what is known about the complex process of learning a second language. To a large extent, the process is similar to that of learning one's first language (Aiken, 1972). The process begins with a period of listening to the sounds and words of the language, followed by attempts at speaking. The learner's initial oral efforts show faulty grammar, incomplete sentences, and errors in pronunciation and vocabulary (Gupta, 1980). During these efforts, the second-language learner is attempting to discover how the new language works and how its syntax or rules organize it. It is not yet clear how the possession of a set of rules for one language affects the generation of rules for a second language.

Second-language learning is made more difficult when the part of the language learned first is the language of textbooks and the classroom. Cummins (1980) has noted the difference between this cognitive academic language proficiency and the language used in social situations. According to Esty & Teppo (1994), the language of textbooks and instruction; "Frequently calls for a high degree of familiarity with words, grammatical patterns, and styles of presentation and arguments that are wholly alien to ordinary informal talk"(p.6). Some of the academic language used in materials and discussions in the mathematics class may be especially difficult for second-language learners to follow. Another critical consideration is the age at which the second language is learned. Language educator used to believe that the younger the learner, the easier a language would be to learn. Recent studies have suggested that the reality is more complicated than this adage suggests. Although older students my learn faster than younger ones, however, they seem to require more formal instruction (or assistance) in the language. The older student is expected to produce a more accurate form of the language, yet the younger student may find more opportunities for interaction with members of the new language group.

Students vary tremendously in the rate at which they learn a second language. Among the factors influencing this rate are one's social and cognitive abilities and one's desire to learn. Some students master the language skills necessary for social interaction in 1 to 3 years. Others, with the same amount of exposure to the second language, take longer. The student who lacks the skills for social interchange, let alone those needed to understand instruction, faces a school experience replete with failure and frustration. Factors other than learner characteristics also influence the learning of a second language. They include the amount of exposure to the new language and the type of language instruction provided. Various aspects of the student's native language and culture can act to facilitate or inhibit learning. This brief account of the process of second-language acquisitions is by no means complete. It is present simply to provide an overview of the problems facing the second-language learner.

Language and Mathematics Learning

The interest in the relationship between language and learning in general is not new. Some the theorists (e.g., Whorf, 1956) have suggested that language determines and defines thought. Others (e.g., Piaget, 1926, 1952; Vygotsky 1962) have tended to accept only a limited effect of language on thought stressing the role of prior cognitive learning in language development and the shifting meanings of words as concepts continue to develop. Although researchers have long recognized the vital role that language plays in mathematics performance (Aiken, 1972), they have not always acknowledged its equally important role in the process of acquiring mathematical concepts and skills. Linguists use the term language register to refer to the meanings that serve a particular function in the language, as well as the words and structures that convey those meanings. A mathematics register, therefore, can be defined at the meanings belonging to the natural language used in mathematics. A mathematics register is more precise that the natural language itself because the meanings of the terms are much narrower in scope. Mathematical terms give rise to "an almost totally non-redundant and relatively unambiguous language" (Brunner, 1976, p.209). Halliday (1975) has suggested that a mathematics register has the following components:

1. Natural language words reinterpreted in the context of mathematics, such as set, point, field, column, sum, even (number), random.
2. Locutions, such as square on the hypotenuse and least common multiple.
3. Terms created from combinations of natural language words, such as feedback and output.
4. Terms formed from combining elements of Greek and Latin words, such as parabole, denominator, coefficient, and asymptotic.

In addition to vocabulary, a mathematics register also includes styles of meaning and ways of presenting arguments within the context of mathematics. These processes require new structures, which are most often borrowed from specialized forms in the natural language. Examples of expressions adapted from English include:

"the area under the given curve"

"the sum of the first n terms of the sequence"

Language Development and Mathematics

Knight and Hargis (1977, p.424) contend that children's language development is likely to affect their mathematics learning. For example, mastery of the grammar of one-to-one correspondence leads to the concept of "manyness." Another set of patterns occurs in noun phrases that "contain the fundamental language vehicle for presenting arithmetic concepts." Finally, these researcher point out that an understanding of the syntax of comparative construction is essential to coping with arithmetic reasoning problems.

Language Concepts of Mathematics

Mathematics language is precise and frequently contains high concepts density, a few words and symbols often convey a very complex idea. In this sense, mathematical statements resemble logical expressions more than ordinary prose. In general, the language of mathematics are within five levels:

- letters
- words (vocabulary)
- sentences
- paragraphs
- discourse

In this hierarchic model, language moves from (small units to complete pieces of text or discourse. To most of us, the language of mathematics is like a foreign language, that 1 a second language with its own grammar, syntax, vocabulary, word order, synonyms, negations, conventions, abbreviations, sentence structure and paragraph structure.

As teacher, educator or mathematician, we often oversee the difficulty face by our students in understanding letters and words, in interpreting mathematical sentences and paragraphs or in understanding the process of problem solving. Nevertheless in every since of manipulating the language of mathematics However we could be completely serious when we assert that mathematics is a foreign language to our students that basic concepts of the language at different level should be made clear to our students before we could expected them to perform satisfactory. Furthermore, language shapes thought. The language of mathematics not only facilitates expression of mathematics thought, including those modes of thought that are essential mathematics concepts. It indeed need to be learned only once and is then good forever after. Fluency in it provides access to the whole world of mathematics.

What are the essential language concepts of mathematics that our students bought to master at high school through college level? Could we teach these concepts in a normal mathematics class?

To answer the above questions, I like to share with you some of the very useful knowledge and information's on language concepts of mathematics exhibited by Esty (1992) the fact that our students are able to 'do' mathematics without 'understanding' its basic logic and concepts worry us. These students of high rate will hit a wall when they are admitted to higher education. As an educator, shouldn't we have the responsibility to teach our students the basic language concepts of mathematics to enable them to 'see', to 'read', to 'write' and to 'discuss' mathematics with the language mathematics or logic, than just limited reasoning.

According to Esty (1992), reading comprehension and writing skills of mathematics require the mastering of logic than just reasoning. This means to be able to 'speak' the language of mathematics we ought to learn about the language concepts or logic of the subject. The issue of whether or not mathematics is a language has received attention from time to time. Although they admitted that there is some validity in the statement that mathematics is a formalized language, Austin and Howson (1979) maintained that it is not a language has its own grammar, syntax, vocabulary, word order, synonyms, negations, conventions, abbreviations, sentence structure, and paragraph structure. Children often find that mathematical text is difficult to read and understand because there are more concepts per word, per sentence, and per paragraph than in texts in other subjects. Reading mathematics is complex because of the mixture of words, numerals, letters, symbols, and graphics that require the reader to shift from one type of vocabulary to another. This clearly indicates that specialized instruction in the reading of mathematics is needed by many learners. Language plays an integral role in the processing of concepts, including mathematical concepts. Effective strategies which assist students to integrate both language and mathematical skills can help the students function more efficiently as independent problem solvers.

In spite or the 'simplicity' of solving the above mathematics problems our students made 'mistake'! What've gone wrong with our teaching? Well, the reason is simple because we have not taught our students how to read with comprehension, to express mathematical thoughts clearly, to reason logically, and to recognize and employ common patterns of mathematical thought. We have not taught them the language of mathematics. Noraini Idris (1999) interviewed six students, and in the interviews each was asked to define and illustrate the meaning of nine mathematical words using: (a) their owns words; (b) a formal mathematics definition (which is employed in mathematics textbooks); (c) diagrams, symbols, or any representations, and/or (d) example(s). All nine mathematical words were from the geometry side of mathematics: perimeter, square, rectangle, circle, area, triangle, isosceles, scalene, and equilateral. In assessing students' understanding of mathematical words, the researcher adopted Otterbum and Nicholson (1979) classification of students responses (see Table 1).

Table 1: Classification of Students' Responses

Classification	Description
Correct	Demonstrates clear knowledge of what the word means
Neglected	Is able to use a word in certain mathematical situations, but is not able to illustrate its meaning clearly and directly
Confused	Has a generally muddles comprehension
Blank	The student does not give any indication that she/he knows the word (silence) or just shakes her/his head.

The results showed that the students' understanding of some of the mathematical words was quite limited. The interviews suggested that the words perimeter, rectangle, triangle and circle were better known than the other five words. Table 2 indicates that four words caused difficulty.

Table 2

Frequency of Students' Understanding of Mathematical Words

Word	Correct	Neglected	Confused	Blank
Perimeter	5	0	1	0
Square	4	0	2	0
Rectangle	6	0	0	0
Circle	6	0	0	0
Area	31	2	0	
Triangle	41	1	0	
Isosceles	22	1	1	
Scalene	22	2	0	
Equilateral	31	2		

So far in this paper the emphasis has been on the effects of verbal factors in learning in mathematics. As Madden (1966), Ausubel and Robinson (1969), Cooper (1971) and other educational researchers have pointed out, however, mathematics itself is a special formalized language and should therefore be taught as such adequately descriptive set of mathematical notations.

Language Analogies to Mathematics

Although there is no one-to-one correspondence between the concepts and rules of mathematics and those of native languages, there are many similarities between verbal and mathematical languages. One teacher (Capps, 1970) has found that pointing out analogies (for example, commutativity, associativity, distributive property) between verbal language and mathematics is a useful instructional technique. It has been noted that an experience-language approach to numbers consisting of eight overlapping stages: (1) engaging in multisensory problems situations; (2) acquisition of oral language to represent in complete sentence form the quantitative relations in, problem situations;(3) introduction of written arithmetic symbols as a shorthand way of writing already known spoken words; (4) acquisition of meaning of written or spoken arithmetic symbols by representing something in experience; (5) after learning to read them, the writing of numbers, number combinations, algorisms, and so on.; (6) acquisition of computational processes by manipulation and discovery, not by memorizing and applying math rules; (7) teaching rules, principles, and generalizations by the inductive-deductive method; (8) continuous interrelationships between first-hand quantitative experiences in life, expression of these in oral and written symbolism, and increasing consciousness and knowledge of the nature of arithmetic.

Language Influences on Mathematical Development

Many writers have referred to various aspects of the interaction between language development and the growth of mathematical understanding.

The importance to mathematical ability of language development has been considered by many psychologists, foremost among whom are Piaget (1954), Bruner (1966), and Galperin (see Lovell, 1971). Piaget maintains that growth in linguistic ability follows the development of concrete operational thought rather than preceding it, although language is important in the completion of such cognitive structures. In contrast, Bruner and his associates (Bruner, Olver, & Greenfield, 1966) maintain that the development of adequate terminology is essential to cognitive growth. Pertinent to the Piaget-Bruner debate, the finding of Olver and Weisberg (1970) that the spontaneous verbalizations of young children are unrelated to their problem-solving performance certainly casts doubt on the directive function of overt speech. Further empirical support for the proposition that verbal ability facilitates the transition from non-conservation to conservation was obtained by Peters (1970). In a study of 131 kindergarten children of low socioeconomic status, verbal training was found to be significantly more effective than non-cued, visual-cued, or no training when the criterion was immediate learning. When the criterion was delayed retention, both verbal training and visual-cued training had greater effectiveness than the other two procedures.

Whether the acquisition of language is a cause or an effect of cognitive development, or, as appears more likely, a bit of both, needs further investigation. Carefully designed studies of the interactions among age, various measures of verbal ability (both overt and covert), general intelligence, and other organismic variables in their effects on the development of the concept of numerosity, the conservation of number and quantity, and other aspects of mathematical knowledge should provide useful information.

Stages in Learning Mathematics: Implications for Instruction

Perhaps the most cogent summary of the instructional implications of stages in mathematical learning has been given by Ausubel and Robinson (1969). These writers begin by pointing out that, at least in the early stages, mathematics deals with concepts, the meanings of which are conveyed by simple explicit images. A second characteristic of mathematical learning is that its operational terms also have explicit, dynamic, or kinesthetic images obtained from the child's experience. A third aspect of mathematical learning is that the child must understand systems of propositions. Ausubel and Robinson maintain that practice in manipulating concrete objects, as in present-day arithmetic instruction, is consistent with the idea that kinesthetic images serve as a basis for understanding arithmetical ideas in particular and the inductive process of concept formation in general.

In a section on learning algebraic symbols and syntax, Ausubel and Robinson (1969) state that the same problems as in learning a second language are involved. The learner begins by translating algebraic symbols into the "native" language of arithmetic and depends on his knowledge of arithmetic syntax in order to understand the syntax of algebra. This is not so simple, because the symbols of algebra bear a one-to-many rather than a one-to-one correspondence to arithmetic symbols. Finally, with repeated application the student reaches a point where the mediational role of arithmetic is no longer needed and he can understand the meaning of an algebraic statement directly. By way of illustration, in learning to understand how the equation 2x + 3 = 11 is solved, the learner obviously needs to know what "2X" and "equation" mean. Furthermore, he must also understand how the understand the propositions that "if equal amounts are added to, or subtracted from, each side of an equation the equality remains" and " if both sides of an equation are multiplied or divided by the same amount the equality remains." Rules such as these can be learned by induction (discovery learning) or by teacher explanation (reception learning). Ausubel, like Gagne (1968), is an advocate of careful sequencing of educational experiences. He stresses the notion that the need for discovery by the learner can be removed by the teacher's meaningful organization of the material to be learned, in addition to overlearning of the part of the student of such sequentially arranged lessons. This approach contrasts with the "discovery learning" advocated by Bruner (1966) and several other writers referred to earlier.

Finally, Ausubel and Robinson (1969) observe that the school is in a much better position with regard to mathematics instruction than it is with language teaching. In the case of language learning, the effectiveness of the parents' (and others) prior verbal interactions with the child plays a crucial role in the latter's understanding of vocabulary and syntax. If the parents' own command of natural language is poor, then many of the linguistic habits of the child may need revising at the outset of his school experiences. The fact that during this period the child continues to be exposed to improper linguistic models at home makes the language teacher's task an unenviable one. On the other hand, parents do not usually teach their preschool children much mathematics beyond rote counting. Therefore, teachers can build on the direct experiences of these mathematically uninstructed children with the physical environment without having to counter the effects of so much ineffective preschool instruction in mathematics.

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